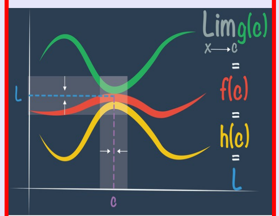
Calculus I

Lecture 23



Feb 19-8:47 AM

Class QZ 19 Domain $(-\infty,\infty)$

Given $f(x)=x^3-3x$

1) Find $f'(x)$, Solve $f'(x)=0$

$f'(x)=3x^2-3$ $3x^2-3=0$ $x^2-1=0$ $\boxed{x=\pm 1}$

$f(1)=-2$

$f(-1)=2$

$f(0)=0$

2) Find $f''(x)$, Solve $f''(x)=0$

$f''(x)=6x$ $6x=0$ $\boxed{x=0}$

3) Do the Sign chart.

x	$-\infty$	-1		0		1	∞
$f'(x)$	+		−		−		+
$f''(x)$	−		−		+		+
$f(x)$	CD		CD		CU		CU

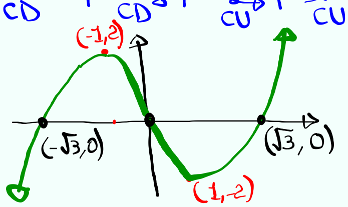
$x^3-3x=0$

$x(x^2-3)=0$

$x=0$

$x^2-3=0$

$x=\pm\sqrt{3}$



May 7-11:07 AM

Class Quiz 20

$f'(x)=0 \to x=1$, und at $x=0$ $\quad$ $f''(x)=0 \to \cancel{x=-1}$, und $\to x=0$

Given $f(x)=(x-3)\sqrt{x}$, $f'(x)=\dfrac{3(x-1)}{2\sqrt{x}}$, $f''(x)=\dfrac{3(x+1)}{4x\sqrt{x}}$

1) Give domain in interval notation $[0,\infty)$

2) Give its Y-Int.

$(0,0)$

3) Give all X-Ints.

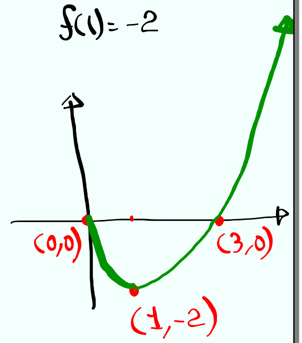
$y=0 \quad (x-3)\sqrt{x}=0$

$(3,0)$, $(0,0)$

4) Make a Complete Sign Chart.

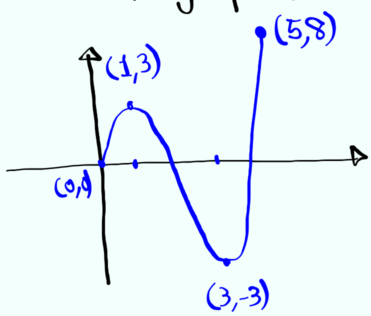
x	0		1		∞
$f'(x)$	o	$-$	•	$+$	
$f''(x)$	o	$+$		$+$	
$f(x)$		↘		↗	

$f(1)=-2$



May 12-7:44 AM

Consider the graph below



Horizontal tan. at

$x=1,\ x=3$

$f'(x)=0 \to x=1,\ x=3$

on the interval $[0,5]$

$f(0)=0$, $f(5)=8$

$f(1)=3$ $\quad f(3)=-3$

Abs. Min. is -3

Abs. Max is 8

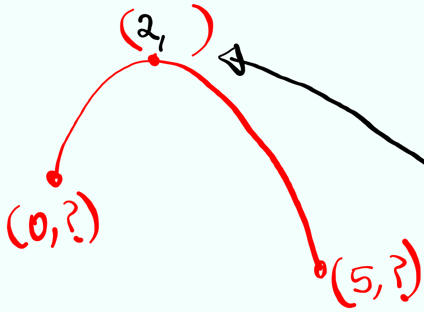
Abs. Max or Min happens at endpoints or at Critical numbers $f'(x)=0$ or undefined.

$f(x)$ has to be Continuous to discuss Abs. Max or Min.

May 12-9:17 AM

$f(x) = 12 + 4x - x^2$ over $[0,5]$

Parabola, opens downward



$f(0) = 12$

$f(5) = 12 + 4(5) - 5^2 = 7$

$f'(x) = 0$

$4 - 2x = 0 \quad x = 2$

$f(2) = 12 + 4(2) - 2^2 = 16$

$f(0) = 12$
$f(2) = 16$
$f(5) = 7$

Abs. Min. Value $7 \to (5,7)$

" Max " $16 \to (2,16)$

May 12-9:23 AM

find Abs. Max & Abs. Min of f

$f(x) = 2x^3 - 3x^2 - 12x + 1$ over $[-2,3]$.

Polynomial

Cont. $(-\infty,\infty)$

$f(-2) = -3$, $f(3) = -8$

$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 0$

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$

$x = 2 \qquad x = -1$

$f(-1) = 8$

$f(2) = -19$

Abs. Max. Value 8

" Min. " -19

May 12-9:28 AM

Find abs. Max & abs. Min of $f(x) = \dfrac{x}{x^2 - x + 1}$

on $[0,3]$.

$x^2 - x + \frac{1}{4} - \frac{1}{4} + 1$

$\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Deno. $\neq 0$

$\boxed{f(0) = 0}$

$\boxed{f(3) = \frac{3}{7}}$

$\left(\frac{-1}{2}\right)^2 = \frac{1}{4}$ Completing the sqr.

$$f'(x) = \frac{1(x^2 - x + 1) - x(2x - 1)}{(x^2 - x + 1)^2}$$

$$= \frac{x^2 \cancel{-x} + 1 - 2x^2 \cancel{+x}}{(x^2 - x + 1)^2} = \frac{1 - x^2}{(x^2 - x + 1)^2} \rightarrow 1 - x^2 = 0$$

$x = 1$, $\cancel{x = -1}$

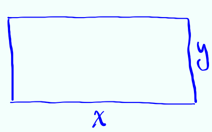
$\boxed{f(1) = 1}$

Abs. Min @ $x = 0 \Rightarrow 0$

Abs. Max @ $x = 1 \Rightarrow 1$

May 12-9:38 AM

Find dimensions of a rectangular shape with area $1000\ m^2$ and Smallest perimeter.



$xy = 1000 \rightarrow y = \frac{1000}{x}$

$P = 2x + 2y$

$P = 2x + \frac{2000}{x}$

$x > 0$, $y > 0$

$f(x) = 2x + \frac{2000}{x}$

$f(x) = 2x + 2000x^{-1}$

$f'(x) = 0$ or und.

$2 - \frac{2000}{x^2} = 0$

$2x^2 - 2000 = 0$

$x^2 - 1000 = 0$

$\boxed{x = 10\sqrt{10}}$

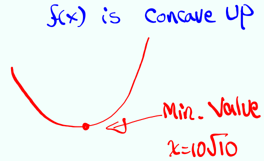
$f'(x) = 2 - 2000x^{-2}$

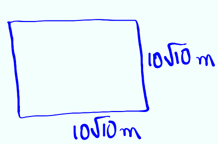
$f''(x) = 4000x^{-3}$

$f''(x) = \frac{4000}{x^3}$

$f''(10\sqrt{10}) = \frac{+}{+} > 0$

$f(x)$ is Concave up





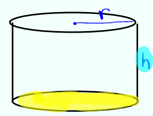
$y = \frac{1000}{x} = \frac{1000}{10\sqrt{10}}$

$= \frac{100}{\sqrt{10}} = 10\sqrt{10}$

May 12-9:50 AM

A cylindrical container with open top has a volume of $1000\pi^4$ ft^3.

Find dimensions that minimizes the cost of metal to make it.



$V = \pi r^2 h$

$1000\pi^4 = \pi r^2 h \rightarrow h = \frac{1000\pi^3}{r^2}$

Materials needed.

$\underbrace{\pi r^2}_{\text{Bottom}} + \underbrace{2\pi r h}_{\text{Side}}$

$f(r) = \pi r^2 + 2\pi r \cdot \frac{1000\pi^3}{r^2}$

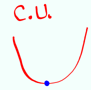
$f(r) = \pi r^2 + \frac{2000\pi^4}{r}$

$f(r) = \pi r^2 + 2000\pi^4 r^{-1}$

$f'(r) = 2\pi r - 2000\pi^4 r^{-2}$

$f''(r) = 2\pi + \frac{4000\pi^4}{r^3} > 0$

C.U.



$f'(r) = 0$

$2\pi r - \frac{2000\pi^4}{r^2} = 0$

$2\pi r^3 - 2000\pi^4 = 0$

$r^3 = \frac{2000\pi^4}{2\pi}$

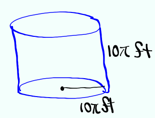
$r^3 = 1000\pi^3 \rightarrow \boxed{r = 10\pi}$

$r = 10\pi$

$h = \frac{1000\pi^3}{(10\pi)^2}$

$h = \frac{1000\pi^3}{100\pi^2}$

$\boxed{h = 10\pi}$

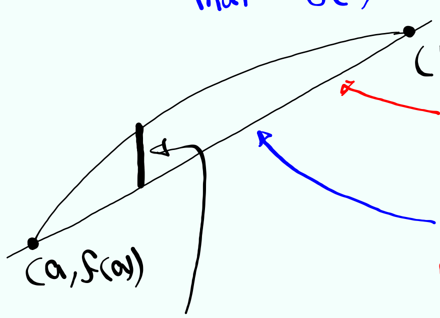


May 12-10:02 AM

$f(x)$ is cont. on $[a,b]$

$f(x)$ is diff. on (a,b)

MVT $\Rightarrow$ there is a number c in (a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$



$m = \frac{f(b)-f(a)}{b-a}$

$y - f(a) = \frac{f(b)-f(a)}{b-a}(x-a)$

$$\boxed{y = \frac{f(b)-f(a)}{b-a}(x-a) + f(a)}$$

Vertical distance between curve & line

$H(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$

May 12-10:20 AM

$H(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$

$H(x)$ is cont. on $[a,b]$

$H(x)$ is diff. on (a,b)

$H(a)=0$, $H(b)=0 \Rightarrow H(a)=H(b)$

By Rolle's thrm

$H'(c)=0$

$H'(x) = f'(x) - \frac{f(b)-f(a)}{b-a} \cdot 1$

$H'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$

$f'(c) - \frac{f(b)-f(a)}{b-a} = 0$

$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$ Conclusion of MVT

May 12-10:25 AM

$f'(x) = 5x^4 - 3x^2 + 4$ $\quad f(-1)=2$

Find $f(x)$.

$f(x) = x^5 - x^3 + 4x + C$

$f(-1) = \cancel{(-1)^5} - \cancel{(-1)^3} + 4(-1) + C = 2$

$-4 + C = 2$ $\quad \boxed{C=6}$

$\boxed{f(x) = x^5 - x^3 + 4x + 6}$

May 12-10:31 AM

$f''(x) = -2 + 12x - 12x^2$ $f(0) = 4$, $\boxed{f'(0) = 12}$

Find $f(x)$.

$f'(x) = -2x + 6x^2 - 4x^3 + C$

$f'(0) = -2(0) + 6(0)^2 - 4(0)^3 + C = 12$ $\boxed{C = 12}$

$f'(x) = -2x + 6x^2 - 4x^3 + 12$

$f(x) = -x^2 + 2x^3 - x^4 + 12x + C$

$f(0) = -0 + 0 - 0 + 0 + C = 4$ $\boxed{C = 4}$

$$\boxed{f(x) = -x^2 + 2x^3 - x^4 + 12x + 4}$$

May 12-10:36 AM

1) Find $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}dx$

$= \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

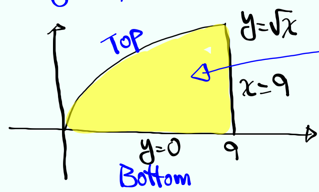
$= \dfrac{x^{3/2}}{3/2} + C = \boxed{\frac{2}{3}x\sqrt{x} + C}$

2) Evaluate $\int_0^9 \sqrt{x}\,dx = \frac{2}{3}x\sqrt{x}\Big|_0^9$

$= \frac{2}{3}\left[9\sqrt{9} - 0\sqrt{0}\right] = \frac{2}{3}\cdot 27$

$= \boxed{18}$

3) Find the area enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 9$.



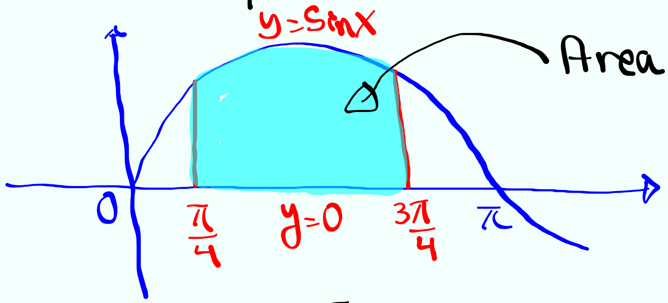
$A = \int_0^9 \left[\overset{\text{Top}}{\sqrt{x}} - \overset{\text{Bottom}}{0}\right]dx$

$= \boxed{18}$

May 12-10:43 AM

Find the area below $y = \sin x$, above x-axis

from $x = \frac{\pi}{4}$ to $x = \frac{3\pi}{4}$.



$$\text{Area} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} [\underset{\text{Top}}{\sin x} - \underset{\text{Bottom}}{0}]\, dx$$

$$= \int_{\pi/4}^{3\pi/4} \sin x \, dx$$

$$= -\cos x \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = -\left[\cos\frac{3\pi}{4} - \cos\frac{\pi}{4}\right]$$

$$= -\left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right] = \boxed{\sqrt{2}}$$

May 12-10:52 AM

Average value of $f(x)$ over $[a,b]$ is

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$f(x)$ must be cont. on $[a,b]$.

Find f_{ave} of $f(x) = 3x^2$ on $[1,2]$.

$$f_{ave} = \frac{1}{\underset{b}{2} - \underset{a}{1}} \int_{\underset{a}{1}}^{2 \leftarrow b} \underset{f(x)}{3x^2}\,dx = \frac{1}{1}\cdot x^3 \Big|_1^2$$

$$= 2^3 - 1^3 = \boxed{7}$$

May 12-10:57 AM

Find f_{ave} of $f(x) = x^3$ over $[-2, 2]$.

$a = -2$, $b = 2$

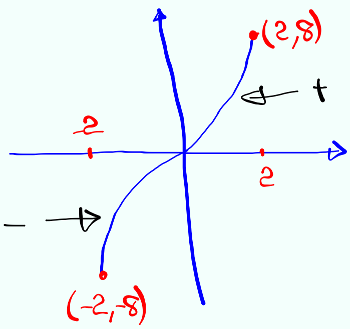
Polynomials

Cont. $(-\infty, \infty)$

$$f_{ave} = \frac{1}{2-(-2)} \int_{-2}^{2} x^3\,dx = \frac{1}{4} \cdot \frac{x^4}{4} \Big|_{-2}^{2}$$

$$= \frac{1}{16}\left[2^4 - (-2)^4\right] = \frac{1}{16}\left[16 - 16\right] = \boxed{0}$$

Ave.



$f(x) = x^3$

$f(-x) = -x^3$

$f(-x) = -f(x)$

odd function

Sym w/t origin.

May 12-11:01 AM

Make sure to do this by Wednesday:

$f(x) = (x-5)\sqrt[3]{x^2}$

$f'(x) = \dfrac{5(x-2)}{3\sqrt[3]{x}}$

$f''(x) = \dfrac{10(x+1)}{9x\sqrt[3]{x}}$

1) Domain

2) All intercepts

3) Find all critical points

4) Find all inflection points

5) Do a complete sign chart

6) Graph $f(x)$.

May 12-11:08 AM